Application of a generalized plasticity constitutive model to a saturated pyroclastic soil of Southern Italy

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Abstract. The paper explores the capability of the advanced Pastor-Zienkiewicz constitutive model to accurately simulate the deformability, the shear strength and the static liquefaction of an air-fall volcanic (pyroclastic) soil, which was involved in catastrophic landslides of the flow type. The calibration and validation of two distinct versions of the PZ constitutive model is done for both undisturbed (loose) and remoulded (dense) saturated specimens. Either drained or undrained triaxial tests are properly reproduced through the model. The potentialities deriving from the use of an advanced constitutive model are pointed out.

Keywords. constitutive, pyroclastic soil, coupling, instability, dilatancy.

1. Introduction

In landslide susceptibility analysis, a relevant issue is the proper modelling of slope failure or soil mechanical instability. A particular case study is represented by the flow-type landslides, for which relative density ($D_r$) and mean effective stress ($p'$) vary significantly from failure to post-failure stage [1]. For instance, a dramatic reduction of the soil shear strength can occur for static liquefaction. In this and similar cases, the hydro-mechanical coupling between the solid skeleton and the pore water pressure plays a paramount role [1]. Significant examples have been recorded for volcanic soils of different Countries, such as those surrounding the Vesuvius volcano [2].

For this class of phenomena, the use of an advanced constitutive model is required to properly reproduce the evolution of the pore water pressures and the modification of the shear strength upon deformation. It entails that the chance to simulate either the transition from a slide to a flow [2] or the build-up of pore water pressures in the post-failure stage really depends on the soil constitutive model used, in addition to the slope geometry, initial and boundary conditions. Moreover, the use of an advanced constitutive model and of a hydro-mechanical coupled finite element model (FEM) can be useful to analyze the post-failure stage of flow-type landslide [1].

In the paper, the Generalized Plasticity constitutive PZ (Pastor-Zienkiewicz) model was dealt with. In the original PZ model the soil relative density is assumed as constant.

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A natural air-fall volcanic (pyroclastic) soil of Southern Italy was selected, for which an extensive data-sets available [6], [7], [8], [9]. Either undisturbed (loose) or remoulded (dense) specimens were examined, which were collected from shallow deposits, 1 to 5 m thick, covering steep slopes (30-40°), prone to catastrophic landslides of the flow-type [10], [11], [12]. The PZ model was applied to simulate the mechanical behavior of the specimens in saturated condition.

This paper discusses the capability of the models to simulate drained or undrained triaxial tests, isotropically consolidated of natural volcanic soil samples. The numerical results were compared to the experimental evidence and some preliminary conclusions were pointed out about the potentialities of the advanced constitutive modelling.

2. The generalized plasticity Pastor - Zienkiewicz constitutive model

So far, many constitutive models have been proposed to reproduce the soil mechanical behaviour. For instance, the Cam Clay model [13] correctly reproduces the behaviour of saturated clays soils; the Hardening Soil Model [14] has been profitably used for dry or saturated sands in drained conditions. Conversely, for coarse-grained collapsible soils the available models are less and those capable to correctly reproduce the hydro-mechanical coupling between the solid skeleton and the pore water pressures are even less. Among these, the PZ model is particularly suitable to describe the behavior of either loose or dense granular soils, both in drained and undrained conditions, even along complex stress paths.

The PZ model derives from the theoretical fundamentals of the Generalised Plasticity Theory [3]. It is assumed that plastic deformations may occur upon either loading or unloading and are derived without the need to define the: i) yielding surface, ii) plastic potential surface, iii) consistency law. The model is completely defined once the following quantities are fixed: i) three directions (load direction \( n_{gL} \) and unload \( n_{gU} \)), ii) two scalars (plastic moduli \( H_L \) and \( H_U \)) and iii) the elastic tensor \( D \).

Globally, 12 parameters are defined (\( G_0, K_0, M_g, M_f, \alpha_g, \alpha_f, H_{0}, H_{m}, \beta_1, \beta_0, \gamma, \gamma_0 \)), which can be calibrated through standard triaxial tests according to the procedures indicated by Zienkiewicz et al. [15]. It is worth noting that the tangential stiffness \( (G_0) \) was obtained by the initial slope angle of the deviatoric stress \( (q) \) versus deviatoric strain \( (\varepsilon_q) \) plot in drained triaxial tests and the volumetric stiffness \( (K_0) \) was calibrated from the results of isotropic compression tests or of drained triaxial tests. The initial Plastic modulus \( (H_0) \) was determined by fitting the experimental curves \( p-\varepsilon_p \) or \( q-\varepsilon_q \). The transition point between the contractive and the dilatative behaviour depends on the ratio between \( M_g \) and \( M_f \).

The former \( (M_g) \) is the slope of the failure criterion in the \( q-p' \) plane, while the latter \( (M_f) \) was derived from the Eq. 1 [3], where \( D_i \) is the relative density. The dilatancy \( (d) \) comes from the Eq. 2, where \( \eta \) is the ratio of the deviatoric stress and the mean effective stress and \( \alpha_f \) is a model parameter to be calibrated.

\[
M_g = M_f D_i \tag{1}
\]

\[
d = (1 + \alpha_f)(M_g - t) \tag{2}
\]
A modification of the PZ model (MPZ) was recently proposed [4], [5] to simulate – with only a set of parameter – the mechanical behavior of sands subjected to large variations of both relative density ($D_r$) and confining pressure ($p'$). In this modified version of the PZ model, the state parameter ($\psi$) proposed by Been and Jeffries [16] was introduced, which measures the vertical distance from the CSL in $e$-$p'$ plane, i.e. the difference between the current void ratio ($e$) and the critical void ratio ($e_{\text{crit}}$) at the same mean effective pressure. Thus, the first ingredient of the “modified model MPZ” is the equation for the Critical State Line, which is taken from Li and Wang [17]. It is reported in Eq. 3, where $e_f$ is the void ratio at $p'=1$ kPa, $\lambda$ and $\xi$ are material parameters.

$$ e_{\text{crit}} = e_f + \lambda \left( \frac{p}{p_{\text{am}}} \right)^q (3) $$

In the modified model MPZ, the parameter $M_f$ is assumed as a function of the relative density and the slope of the Critical State Line (Eq. 4), where $h_1$ and $h_2$ are two constants to be calibrated. The Plastic modulus ($H_0$) is computed from Eq. 5.

$$ M_f = h_1 - h_2 \psi_f, \quad \psi_f = \left( \frac{e}{e_{\text{crit}}} \right) $$

$$ H_0 = H' \exp \left[ - \beta \left( \frac{e}{e_0} \right)^\delta \right] (5) $$

$$ d = \frac{d_n}{M_s} \left( M_s \exp^{n_r - \eta} \right) (6) $$

3. Experimental tests

The PZ Model was tested for a pyroclastic soil of Southern Italy. The soil specimens were derived from a specific site (Santa Lucia, Pizzo d’Alvano massif, 40°49’53” N, 14°38’10” E), where huge flowslides occurred in May 1998 [10].

The specimens were either undisturbed (loose) or remoulded (dense) and they belong to the Class “A” defined by Bilotta and Foresta [19]. This is the class of the finest ashy soils detected in the pyroclastic soils district, which is 3’000 km² large [20]. The undisturbed ashy A soil is characterized by high porosity, low specific gravity ($G_s$ average value is 2.51) because of voids internal to the solid particles, low dry unit weight ($\gamma_d$), and a metastable structure [8]. Conversely, the remoulded specimens show an initial void ratio ($e_0$) lower than the undisturbed material, and a dilative behavior.

The investigated pyroclastic soil shows an intermediate strain behavior between granular and cohesive material [7]; it means that like cohesive materials they are remarkably deformable for stress path which are distant from the failure, while it is not possible to locate only one NCL, like for granular materials.

In CID (Consolidated Isotropically Drained) compression triaxial tests (Fig. 1a-c) the undisturbed specimens show a hardening behavior and don’t reach any critical state,
so it is difficult defining the position of the Critical State Line (CSL). Therefore, for each test, the maximum deviatoric stress \((q)\) was extrapolated at a 40% distortional strain \(\varepsilon_q\) to draw the CSL. Doing so, and assuming nil cohesion \((c')\), \(M_g\) was estimated equal to 1.55, which corresponds to a CSL slope angle \(\lambda\) equal to 0.237.

In the CIU (Consolidated Isotropically Undrained) tests (Fig.1d-f), the undisturbed (loose) specimens have an unstable behavior (e.g. test BIS03_05), while the remolded (dense) specimens show a stable hardening behavior (e.g. test BIS18_06).

![Figure 1. CID triaxial tests on undisturbed specimens (on the left) and CIU triaxial tests (on the right) on undisturbed and remoulded specimens of ashy A soil (data from Lancellotta et al., 2012).](image)

4. Constitutive modelling

4.1. Simulation of drained triaxial tests

Both the original PZ and the modified MPZ model were calibrated for the CID triaxial tests of figure 1a-c. The numerical results were fitted to the experimental data through a trial-and-error procedure, while more sophisticated methods – e.g. inverse analysis – were out of the scope of the paper.

The tests refer to a narrow range of relative density \((0.27<D_r<0.39)\) while spanning over a large interval for the initial confining pressure \((50<p'_{\text{initial}}<150)\). Thus, the PZ model was independently calibrated and validated for distinct initial confining pressures \((50, 100\) and \(150)\). Particularly, \(G_0\) was obtained from the experimental plot \((q-\varepsilon_q)\), while \(H_0\)
Table 1. Calibration and validation of the PZ model and modified model MPZ for drained triaxial tests.

<table>
<thead>
<tr>
<th>Test</th>
<th>Specimen (s):</th>
<th>$p'$ (kPa)</th>
<th>$e_0$</th>
<th>$D_e$</th>
<th>#</th>
<th>$H_0$</th>
<th>$M_f$</th>
<th>$h_1$</th>
<th>$h_2$</th>
<th>$d_2$</th>
<th>$m$</th>
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<tr>
<td>BIS02_05</td>
<td>u</td>
<td>50</td>
<td>2.281</td>
<td>0.27</td>
<td>C</td>
<td>86</td>
<td>0.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BIS09_04</td>
<td>u</td>
<td>50</td>
<td>2.385</td>
<td>0.22</td>
<td>V</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BIS13_04</td>
<td>u</td>
<td>100</td>
<td>2.110</td>
<td>0.34</td>
<td>C</td>
<td></td>
<td></td>
<td>1</td>
<td>0.35</td>
<td>0.28</td>
<td>2.5</td>
</tr>
<tr>
<td>BIS04_05</td>
<td>u</td>
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<td>2.059</td>
<td>0.37</td>
<td>V</td>
<td>110</td>
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<td></td>
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</tr>
<tr>
<td>BIS05_05</td>
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<td>150</td>
<td>2.007</td>
<td>0.39</td>
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<tr>
<td>BIS08_04</td>
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<td>120</td>
<td>0.60</td>
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</tr>
</tbody>
</table>

specimen (s): undisturbed (u), remoulded (r), #: (C) Calibration, (V) Validation

$PZ$ model: $\lambda=0.274$, $\kappa=0.015$, $e_0=2.3kPa$, $M_f=1.55$

modified model $MPZ$: $\lambda=0.274$, $\kappa=0.015$, $e_0=2.3kPa$, $M_f=1.55$

$\psi, \kappa, \lambda$ were calibrated through the best fitting of all the curves of Fig. 1a-c starting from the initial value provided by the Eqs. 1-2.

The calibrated $M_f$ was quite close to Eq. 2.

The calibrated parameters are in Tab. 1, while the experimental evidence and the numerical results are compared in figure 2a-c. On the other hand, the modified model $MPZ$ (Fig. 2d-f) was successfully calibrated and validated for all the 6 tests of figure 1d-f with a single set of parameters, independent on the confining pressures (Tab. 1).
It is worth noting that the interpretation quality (calibration) and the capability prediction (validation) are satisfactory for both the models. Nevertheless, the soil deformability is better captured by the modified model MPZ.

4.2. Simulation of undrained triaxial tests

The CIU triaxial tests of figure 1d-f were simulated through both the original PZ and the modified model MPZ. Here, the goal was to check the capability of a generalized constitutive model to reproduce the global behaviour of a natural soil, either contractive or dilative depending on its own soil structure, respectively for undisturbed (loose) or remoulded (dense) specimens.

The numerical results are provided in figure 3 in terms of \( q-p' \), \( q-\varepsilon_1 \) and \( p_w-\varepsilon_1 \). It is worth noting that both the PZ models are capable to satisfactorily reproduce the experimental results.

For the undisturbed specimens, the parameters calibrated for the original PZ model in undrained tests (Tab. 2) are quite different than those achieved for the drained tests (Tab. 1); this is due to the difference in the relative density \( D_r \) of the specimens as well as to the different stress paths - which differently modify the mean effective pressure \( p' \) during the test -. Conversely, the modified model MPZ was capable to well reproduce either the drained or undrained tests, as well as independent on \( p' \) and \( D_r \), through a unique set of constitutive parameters.

![Figure 3. CIU triaxial tests simulated through the PZ model (left) and modified PZ mode (right).](image-url)
5. Discussion and conclusions

The paper was aimed to verify the capability of the Pastor-Zienkiewicz constitutive model to quantitatively simulate the mechanical behaviour of an air-fall volcanic (pyroclastic) soil, frequently involved in catastrophic flow-like landslides. Particularly, two sets of drained and undrained compression triaxial tests were analyzed.

The original PZ model was successfully calibrated and validated for both types of experimental tests. As the model assumes the soil behaviour dependent on the selected values for $H_0$ and $M_g$, which are related to the mean pressure and relative density, the best fitting of the experimental tests was obtained with different sets of constitutive parameters. Despite this test-specific calibration, the numerical results are appreciable.

For the modified model MPZ, the possibility to individuate a unique set of constitutive parameters, which allow simulating well all the selected (drained or undrained) triaxial tests, for different $p'$ and $D_r$, enhance the possibilities to numerically reproduce realistic stress paths experienced by the soil in boundary value problems. This is because the dilatancy may change in the model in relation to the distance from the critical state line (i.e. proximity to failure).

In the paper, the performances of the constitutive PZ model were simply evaluated through a trial-and-error fitting procedure between the experimental and numerical curves, without any advanced algorithm used. Thus, the result of this procedure is clearly user-dependent. An optimization algorithm could be used as in [21]. Nevertheless, the aim of the paper was to check for the overall applicability of the PZ model for the case of pyroclastic soils. The paper demonstrated that both the versions of the PZ model are able to correctly reproduce either the contractive or the dilative behavior of pyroclastic soils.

It is also worth noting that the performance of the PZ model about the onset of instabilities on loose samples (static liquefaction) is well documented in [1]. For the reproduced experimental tests, the transition from a contractive to dilative behavior is correctly captured and the computed pore water pressures also fit the measurements. In other testing conditions, the opposite transition (from dilative to contractive) would be expected, more interestingly, and the PZ model should be capable to simulate this case. This is one of the future research perspectives, which certainly need to be further investigated through experiments and modeling.
References


